Control Systems with additional loops

doc. Ing. Petr Blaha, PhD.
On-Off Control

One of the most simple and most adopted controllers. Control variable assume just two values $u_{min}$ and $u_{max}$. Selection depends on sign of control error.

$$u = \begin{cases} 
  u_{max} & \text{if } e > 0 \\
  u_{min} & \text{if } e < 0 
\end{cases}$$

Minimum value of control variable is often set to zero $u_{min} = 0$. Main drawback of this type of control are persistent oscillations around the set-point.

Practical realizations usually use relay. Measurement noise causes very fast changes in control variable and thus very high tear. Relay can be modified with dead zone (three-state relay) or with hysteresis to reduce switching.

Cost effective solution. Sometimes called **bang-bang** control ($u_{min} \neq 0$).
Control systems with model of controlled system

Control theory
Circuit with controlled system model

Control transfer function and disturbance transfer function

\[ F_w(p) = \frac{Y(p)}{W(p)} = \frac{R_1(p)S(p) + R_1(p)S(p)R_M(p)M(p)}{1 + R_1(p)S(p) + S(p)R_M(p) + S(p)R_1(p)M(p)R_M(p)} \]

\[ F_u(p) = \frac{Y(p)}{U(p)} = \frac{S(p)}{1 + R_1(p)S(p) + S(p)R_M(p) + S(p)R_1(p)M(p)R_M(p)} \]
Circuit properties

- improved control performance
- control performance is not sensitive to parameter changes in controlled system (controller $R_M(p)$ takes care of it)
Circuit properties

- improved control performance
- control performance is not sensitive to parameter changes in controlled system (controller $R_M(p)$ takes care of it)
- improved control performance
- control performance is not sensitive to parameter changes in controlled system (controller $R_M(p)$ takes care of it)
Enables to compensate the delay term in controlled system

\[ F_w(p) = \frac{Y(p)}{W(p)} = \frac{R(p)S(p)e^{-\Delta p}}{1 + R(p)M(p)} \]

In ideal case \( S e^{-\Delta p} = M e^{-\Delta M p} \) controller controls only model \( M \) without time delay. Characteristic polynomial does not contain term with time delay. This fact improves stability of closed loop system.
Control system with auxiliary control variable
Control system with auxiliary control variable

Block diagram

Closed loop transfer functions

\[ F_w(p) = \frac{Y(p)}{W(p)} = \frac{R_1(p)R_2(p)S_1(p)S_2(p)}{1 + R_2(p)S_1(p) + R_1(p)R_2(p)S_1(p)S_2(p)} \]

\[ F_u(p) = \frac{Y(p)}{U(p)} = \frac{S_2(p)[1 + S_1(p)R_2(p)]}{1 + R_2(p)S_1(p) + R_1(p)R_2(p)S_1(p)S_2(p)} \]
Control systems with auxiliary control variable are used in:

- temperature control
- position servomechanisms
Example of position servomechanism

Properties of control systems with auxiliary control variable

- auxiliary loop influences characteristic polynomial
- faster reaction to disturbances - inner loops takes care of disturbance rejection
- capable to deal with successive integrators without feedback
- simpler work with control action limits (voltage and current limits in servo systems)
Control system with auxiliary manipulated variable
Control system with auxiliary manipulated variable

Closed loop transfer functions

\[ F_w(p) = \frac{Y(p)}{W(p)} = \frac{[R_1(p)S_1(p) + R_2(p)]S_2(p)}{1 + R_2(p)S_2(p) + R_1(p)S_1(p)S_2(p)} \]

\[ F_u(p) = \frac{Y(p)}{U(p)} = \frac{S_2(p)}{1 + R_2(p)S_2(p) + R_1(p)S_1(p)S_2(p)} \]
Solution of temperature control in a control system with auxiliary manipulated variable
Basic properties

- both loops influence stability
- significantly improves disturbance rejection
Basic properties

- Both loops influence stability
- Significantly improves disturbance rejection
Basic properties

- both loops influence stability
- significantly improves disturbance rejection
Disturbance feed-forward compensation on a manipulated variable
Disturbance transfer function

\[ F_u(p) = \frac{Y(p)}{U(p)} = \frac{S_u(p) + R_2(p)S(p)}{1 + R_1(p)S(p)} \]
Invariance against disturbance

Proper selection of $R_2$ can significantly influence disturbance transfer function. In case of its complete cancellation we talk about disturbance invariant system. The disturbance will be fully compensated if

$$S_u(p) + R_2(p)S(p) = 0$$

evaluating $R_2(p)$

$$R_2(p) = -\frac{S_u(p)}{S(p)}$$

Practical realization of this controller is often not possible. Its transfer function is realizable if $S_u(p)$ is of higher or at least equal degree than $S(p)$. This condition only seldom holds in practical cases. The polynomial degree in numerator is higher than in denominator which is impossible to realize.
Basic properties

- compensator design often leads to non-causal system. Often we are satisfied with static invariance against disturbance (in steady state).
- significantly improves disturbance attenuation
- useful for temperature control of large volumes - equitherm regulation of buildings (outdoor temperature measurement)
- compensator design often leads to non-causal system.

- Often we are satisfied with static invariance against disturbance (in steady state)

- significantly improves disturbance attenuation

- useful for temperature control of large volumes - equitherm regulation of buildings (outdoor temperature measurement)
Basic properties

- compensator design often leads to non-causal system. Often we are satisfied with static invariance against disturbance (in steady state)
- significantly improves disturbance attenuation
- useful for temperature control of large volumes - equitherm regulation of buildings (outdoor temperature measurement)
Basic properties

- compensator design often leads to non-causal system. Often we are satisfied with static invariance against disturbance (in steady state)
- significantly improves disturbance attenuation
- useful for temperature control of large volumes - equitherm regulation of buildings (outdoor temperature measurement)
State space representation of dynamic systems
General dynamic system

Control systems with model of controlled system

Cascade control structure

Control theory

Control Systems with additional loops – strana 20 / 7
Basic notations

**State variable** function of time, does not have to really exist in a real system

**State** \( n \) state variables which determine system behavior,

**State vector** column vector \( \mathbf{x}(t) \) whose items are individual state variables

\[
\mathbf{x}(t) = (x_1(t) \ x_2(t) \ \cdots \ x_n(t))^T
\]  

(1)

**State space** \( n \) dimensional space \( \mathbb{R}^n \), state is a point in this space

**Input vector** for system with \( m \) inputs an input vector is

\[
\mathbf{u}(t) = (u_1(t) \ u_2(t) \ \cdots \ u_m(t))^T
\]  

(2)
Basic notations - cont.

Output vector \( r \) outputs collected in column vector

\[
y(t) = \begin{pmatrix} y_1(t) & y_2(t) & \cdots & y_r(t) \end{pmatrix}^T
\] (3)

State equations matrix equations describing behavior of dynamic system

First state equation it defines first derivatives of state as linear combination of states and inputs

\[
\begin{align*}
\dot{x}_1 &= a_{11}x_1(t) + \cdots a_{1n}x_n(t) + b_{11}u_1(t) + b_{1m}u_m(t) \\
\dot{x}_2 &= a_{21}x_1(t) + \cdots a_{2n}x_n(t) + b_{21}u_1(t) + b_{2m}u_m(t) \\
& \vdots \\
\dot{x}_n &= a_{n1}x_1(t) + \cdots a_{nn}x_n(t) + b_{n1}u_1(t) + b_{nm}u_m(t)
\end{align*}
\] (4)
Matrix description

\[ \dot{x}(t) = Ax(t) + Bu(t) \]  \hspace{1cm} (5)

**Second state equation** describes the outputs as a linear combination of states and inputs

\[
\begin{align*}
    y_1 &= c_{11}x_1(t) + \cdots + c_{1n}x_n(t) + d_{11}u_1(t) + d_{1m}u_m(t) \\
    y_2 &= c_{21}x_1(t) + \cdots + c_{2n}x_n(t) + d_{21}u_1(t) + d_{2m}u_m(t) \\
        &\vdots \hspace{1cm} \vdots \\
    y_r &= c_{r1}x_1(t) + \cdots + c_{rn}x_n(t) + d_{r1}u_1(t) + d_{rm}u_m(t)
\end{align*}
\]  \hspace{1cm} (6)

Given in matrix form

\[ y(t) = Cx(t) + Du(t) \]  \hspace{1cm} (7)
Meaning of individual matrices

\[ A \text{ is state matrix (also system matrix or matrix of feedbacks). It has dimension } n \times n \]

\[ B \text{ shows how the system is connected to inputs (also input matrix). It has dimension } r \times n \times m. \]

\[ C \text{ shows how states are connected to output (also output matrix). It has dimension } r \times n. \]

\[ D \text{ shows direct connection of inputs to outputs (also feed forward matrix). It has dimension } r \times n. \text{ From the viewpoint of dynamic behavior this matrix is not important. This matrix is often zero matrix.} \]
Different colors are used for different vector dimensions.
Utilization of state feedback (feedback from state vector) enables to distribute arbitrarily closed loop system eigenvalues (pole placement method).

\[
\dot{x}(t) = Ax(t) - BKx(t) + Bw(t) = [A - BK]x(t) + Bw(t) \\
y(t) = Cx(t)
\] (8)
Utilization of state feedback (feedback from state vector) enables to distribute arbitrarily closed loop system eigenvalues (pole placement method).

\[
\dot{x}(t) = Ax(t) - BKx(t) + Bw(t) = [A - BK]x(t) + Bw(t) \\
y(t) = Cx(t)
\]